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A theoretical and experimental case for state vector physicality

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Abstract: A physical description of the state vector is developed. The role of complex variables in this description is discussed in both a historical context of number set acceptance, and in an examination of inherent mathematical properties. The Afshar experimental results are presented and are demonstrated to give real evidence that the state vector is a standing wave along lightlike paths in Minkowski space time. © *2009 Physics Essays Publication*. [DOI: 10.4006/1.3262492]

Résumé: Une description physique du vecteur d'état est développée. Le rôle des variables complexes dans cette description est discuté dans un contexte historique d'acceptation de groupe de nombre, et dans un examen de leurs propriétés mathématiques inhérentes. Les résultats expérimentaux d'Afshar sont présentés. Ces résultats donnent la vraie évidence que le vecteur d'état est une onde stationnaire le long de chemins de la lumière dans l'espace temps de Minkowski.

Key words: State Vector; Afshar Experiment; Transactional Interpretation of Quantum Mechanics; Real Numbers; Imaginary Numbers; Complex Numbers; Minkowski Space Time.

I. INTRODUCTION

One of the criticisms by Einstein and countless others of the current matrix probability method of quantum mechanics is that it lacks of an overall coherent visualization. A physical model of the state vector would contribute largely to this goal. The imaginary number representation of the state vector and the inability of Schrödinger's wave packet to explain wave front collapse are two main reasons for refuting state vector physicality. These reasons are here investigated and are found to allow for a physical model. A discussion on the mathematical nature of the complex plane and recent evidence from the Afshar experiment lend credence to a modified version of the transactional interpretation's model of the state vector.

II. NAMING AND ACCEPTANCE OF NUMBER SETS

One of the main arguments against state vector physicality is that the state vector contains an imaginary number and, therefore, cannot be a real quantity. By investigating the history of negative numbers and complex numbers, one can show that the term imaginary is a misnomer. This is reinforced by the related terms real and complex. By investigating the functionality and inherent nature of complex numbers one can determine that their proper applied mathematics is a description of time.

One can look at the history of other number sets and see the reluctance to accept them based on the fact that the application of these numbers was not yet known. One can empathize with this reluctance if there is no applied mathematics associated at the time of its development. The original purpose for numbers was simply to count. Negative numbers are absurd for counting oxen; one cannot have −12 oxen. Although negative numbers were used in China, this concept did not begin to appear in the west until well after 628 A.D. when the Indian mathematician Brahmagupta, in Brahma-Sphuta-Siddhanta,[1] referred to negative numbers as debts. Negative numbers finally started to make some sense.

Negatives also do not make sense when constructing architecture. One cannot have a side of length −60. During the third century A.D., Diophantus in *Arithmetica* referred to an equation with a negative solution as absurd. As late as the 18th century, negative numbers were considered as having an evil influence. In A.D. 1759, Francis Maseres wrote that negative numbers "darken the very whole doctrines of the equations and make dark of the things that are in their nature excessively obvious and simple."[2] He came to the conclusion that negative numbers were nonsensical.

However, negative numbers do make sense in terms of debt. They also make sense in terms of a Cartesian space coordinate system and, most importantly, as a defined vector direction. The terms "real numbers" and "imaginary numbers" were originally used for the now ubiquitous DesCartes geometry.[3] It was not until much later that the terms were adopted for complex numbers.

III. INVESTIGATION INTO COMPLEX, REAL, AND IMAGINARY NUMBERS

Many physicists are comfortable working with complex numbers today. They have become so commonplace that they are routinely used in Fourier analysis, Laplace transforms, electrical engineering, quantum mechanics, and relativity. In the cases typically used by engineers, the waves could be wholly represented by sin and cosine functions, but are expressed in the complex plane through Euler's identity for calculative convenience. This is demonstrated in impedance and reactance. The physical quantity does not exist in the complex plane, but does exist in Euclidean space.

This evolved in quantum mechanics. Schrödinger's wave packet theory[4] used the same calculative convenience of

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classical mechanics phase space. Originally the wave function ψ was constructed as $A\cos(kx-wt)$; however, it is calculative convenience if the wave function is of the form $\psi = Ae^{i(kx-wt)}$. Allowing A to be complex simply provides an easy way to handle phase factors between interfering waves. When the amplitude A is real, the intensity is still the square of the amplitude; when A is complex, the intensity is the square of the absolute value of A, or $|A^*A|$, which is still a real number. To describe the experimental results with the actual cosine would cause the wave front to collapse at a velocity greater than c, so a real physical wave that exists in Euclidean three dimensions was not possible. The fact that Schrödinger's wave packet is described using a complex number and cannot be described using a "real" number is the reason given that the wave function, itself, cannot be real. We are instructed not to worry about this fact.[5] Fuchs and Peres[6] stated the case of the Copenhagen interpretation well. "… In classical mechanics phase space points correspond to objective data, whereas in quantum mechanics Hilbert space points correspond to quantum states. The analogy is misleading: Attributing reality to quantum states leads to a host of "quantum paradoxes." (*Hilbert space* is complex.) The word reality is misleading here. It does not consider the possibility of a four-vector standing wave in the complex plane. This would not be real it would be "imaginary." The Copenhagen interpretation rests its reputation on correctly refuting the classical *phase space* of a Schrödinger wave packet, and not on rejecting a four-vector standing wave.

The case of special relativity (SR) is completely different. Here the complex plane is not used for convenience, but is a direct result of solutions to the Lorentz transformation equations. In SR, the complex plane as described by Minkowski space time is necessary. It is used to represent an actual physical model. "A physical entity … is called a 4-vector, … corresponding in their relations of reality and the properties of transformation."[7] Unlike the state vector SV, this common usage of the complex number enjoys almost universal acceptance, despite the fact that it represents actual physical entities.

A. Real and imaginary versus algebraic and geometric

When a real number is squared, there are two types of mathematics that are represented. x^2 can be algebraic or it can be geometric. The algebraic aspect is simple multiplication of real numbers. The geometric aspect is displayed when x^2 denotes two dimensions instead of one. The power represents the dimensionality. We know by connotation whether x^2 is algebraic or geometric. With complex numbers the algebraic and geometric notions are no longer a connotation but naturally separated. The real and imaginary components are here analyzed.

Whenever there is a multiplication by the imaginary component of a complex number, there is always a rotation of the axis. For example, $i(4+5i)$ becomes $4i-5$. This is a 90°CCW rotation and, therefore, a geometric change. One can see that this rotation displays a system of infinite coordinates systems. i is also called the rotation operator.

In contrast, multiplication by the real component of the complex number does not cause a rotation of the vector. This represents the scalar or algebraic part of multiplication. If one multiplies a complex vector by a real number then the complex vector will change its magnitude or direction only. For example, $3(4+5i)$ results in a scalar change to $12+15i$. Hence, this last result can be viewed as representative of the algebraic or scalar properties.

The mathematician Nathan Altshiller Court sums up the misnomer of imaginary numbers.[8]

> Just as the Holy Roman Empire was not holy Roman nor an empire, imaginary points are not imaginary and they are not points…There is nothing imaginary about them except the name…They are obviously not points, since an imaginary point is, by definition, an infinite collection of points. One is reminded of the fact that an irrational number, and even a rational number, may be defined in terms of an infinite number of rational numbers (the Dedekind cut).

One can see from this short review that complex numbers are not a real versus imaginary issue at all, but are better described as an algebraic versus geometric issue. To advance this concept is beyond the scope of this paper; however, the purpose of this exercise is to present the true meaning of complex numbers. It is also presented to demonstrate the confusion that the terms *real* and *imaginary* have caused quantum mechanics when discussing *reality*.

B. The square root function and complex numbers

It is well known that complex numbers are derived from the investigation of the limits of the square root function with regard to negative numbers. An investigation into this operation reveals its potential usefulness further. The square root is an operation defined as follows: "What is the original number, when multiplied by itself gives the presented number?" This definition implies cause and effect: "The presented number was the effect produced by a causal number(s) multiplied by itself." The cause and effect nature of this definition gives the square root an inherent reverse time quality. The square root is unlike other operations. It is, by its very nature, an operation that is *defined as reversed in time*. In my humble opinion, this fact is greatly underappreciated.

What assumptions are wrong in physics today? Smolin[9] stated the following: "… it involves two things: the foundations of quantum mechanics and the nature of time." As the proper applied mathematics for negative numbers was eventually discovered to be debt and vector direction, maybe we should consider the possibility that the proper applied mathematics for imaginary numbers is time. This is reasonable when one considers their inherent timelike properties. This notion is reinforced with Minkowski space time representing real phenomena. Additionally, we have found that the state vector is represented in the time dependent Schrödinger equation by a complex amplitude only. This is also the direct result of observed phenomena. Taken together one can make a cogent case that the state vector as a real physical quantity in the complex plane should not be discounted.

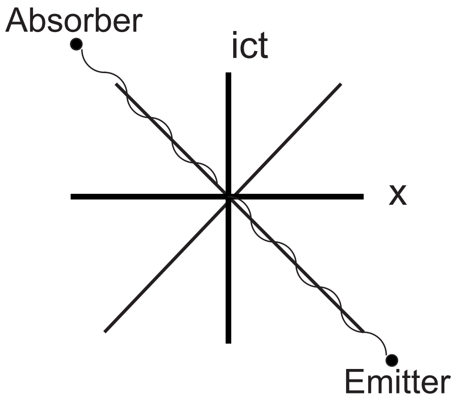

FIG. 1. State vector as a standing wave in Minkowski space time.

IV. EXPERIMENTAL DEVELOPMENT OF A PHYSICAL STATE VECTOR

A. The transactional interpretation and the state vector

The transactional interpretation (TI) of quantum mechanics[10] gives physicality to the state vector. It is consistent with the double slit and other quantum experiments. In TI, the state vector is a standing wave that forms along the lightlike paths in complex Minkowski space time. It is a four-vector (x,y,z,ict) standing wave, which connects an emitter with an absorber.

The common model of light is of a photon/wave that travels at c. This is different in TI. In TI, photon energy is transferred at c when a four-vector standing wave is generated between an emitter and an absorber. The emitter sends out the offer wave ψ and receives confirmation waves ψ^* back from all possible absorbers. This transaction produces the final four-vector standing wave. The four-vector standing wave is the state vector. See Fig. 1. The concept of a four-vector standing wave was first introduced by Wheeler and Feynman.[11]

This four-vector standing wave exists along the positive and negative lightlike paths in Minkowski space time. This makes it inherently nonlocal and atemporal. It therefore does not have a problem with wave front collapse.

The offer wave ψ and the confirmation wave ψ^* overlap and naturally derive the Born probability law. $\langle x\rangle = \int_{\text{vol}} \psi^* X \psi dv$. This is an average over space of the possible values of x, which the operator X projects from the components of the offer wave.

In TI, there is a pseudotime that arises as the offer and advanced waves travel to each other to create the standing wave, which then causes a transaction of energy transfer. This concept is underdeveloped.[12] As with apples, we do not need to throw out the whole bunch or, in this case theory, because of one perceived bad apple of pseudotime. The concept of the state vector as a standing wave between emitter and absorber in space time can stand alone with or without pseudotime.

TI has several advantages over the Copenhagen interpretation.

(1) Since the TI state vector model is nonlocal and atemporal, it does not need to invoke arbitrary collapse triggers such a consciousness, etc., because it is the absorber rather than the observer that precipitates the collapse of the state vector.
(2) TI explains the Wheeler delayed choice experiment, which hints at a standing wave.
(3) The Born probability function is derived from the physical model and not just stated as an axiom as in the Copenhagen interpretation.
(4) Most importantly, TI is experimentally proven to be preferred to the CI and the many-worlds interpretation (MWI) by the Afshar experiment.[13]

V. THE AFSHAR EXPERIMENT

In the famous Bohr–Einstein debates at the 1930 Solvay conference,[14] Einstein proposed several Gedanken experiments to challenge the Copenhagen interpretation of quantum mechanics. It appears from these debates that Einstein, although unsuccessful, was attempting to propose a nondestructive test that would allow conjugate variables to be measured. In fact, later in the EPR paper,[15] the following was stated.

> If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

The debate continues as Fuchs and Peres recently stated the following: "If quantum theory had been in a crisis, experimenters would have informed us long ago."[6] This opinion is open to question once again. The debate now has an experimental test of complimentarity that Einstein was searching for.

The Afshar experiment has successfully proven to violate Bohr's principle of complementarity (BCP). Afshar uses nondestructive testing of wave minima to determine wavelength along with a lens to determine which way information in a double slit welcher-weg experiment. A wire grid is placed in front of the lens where the presence of interference minima should be located. With the spacing of the minima one can calculate the wavelength. Once a photon is detected by a detector on the screen, its trajectory can be traced due to lens focusing and linear momentum. The flux versus screen position is measured for the various testing configurations.

The results are shown in Fig. 2. Configuration (a) displays the setup and results with both slits open and no wire grid. Configuration (b) shows the setup and results with both slits open and a wire grid. Configuration (c) shows the setup and results with a wire grid and hole A blocked. Configuration (d) shows the setup and results a wire grid and hole B blocked. When either slit is individually blocked the wire grid produces scattering and reduces the total flux at the detectors. When no hole is blocked, with or without the wire grid in place, negligible scattering is measured. This demonstrates the presence of interference when both holes are open and the wire grid is present. Since both photon wavelength and visibility are measured to 98%, this violates the

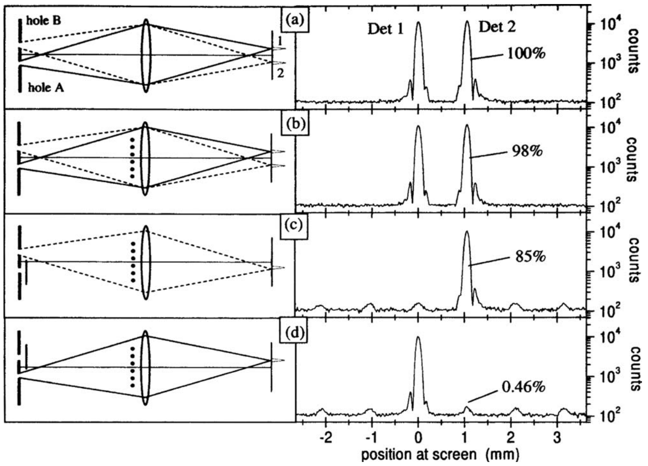

FIG. 2. Afshar experimental results.

Greenberger–Yasin[16] inequality $V^2+K^2 \leq 1$, where V is the normalized wave visibility and K is the normalized particle position. This is the quantification of BCP.

One criticism of the Afshar experiment is that it does not apply to BCP as it utilizes a nondestructive test to determine *wavelength*. This argument applies a double standard and can be discredited. One can see from any accepted textbook description of the welcher-weg double slit experiment that *particle* position in the plane of the slits has always been measured indirectly. Particle position is measured nondestructively by using a light flash from an electron in the vicinity of the slits,[17] tracing the path to the slit using the linear momentum deduced from detector position, and measuring slit size while assuming a particle had to go through either of the apertures. These inferences proceed with as much, if not less, rigor than the wavelength measurement using Afshar's wire grid minima detection method.

Another criticism of the Afshar experiment comes from Kastner.[18] She presented a quantum state analysis of a multiple Stern–Gerlach experiment, which is in many ways analogous to the Afshar double slit setup. She derives the wavelength pattern measured by the wire grid as approximately 1, but the which-way information as 0 and therefore the Greenberger–YaSin inequality is fulfilled with $V^2+K^2 \leq 1$. Essentially, Kastner contended that the lens decomposes the quantum state. Afshar's fellow researchers Flores and Knoessel[19] addressed this criticism in full with an analysis of a modified Afshar experiment. In their setup shown in Fig. 3, the double slit is replaced with two separate coherent laser beams that overlap under a small angle. A wire grid is placed at minima in the overlap region to measure wavelength. The angle of the beams separates them in the far field without the use of a lens system. Which-way information is determined through momentum conservation and a study of the electric fields involved. By removing the lens and the double slit, Flores and Knoessel removed the contention of Kastner.

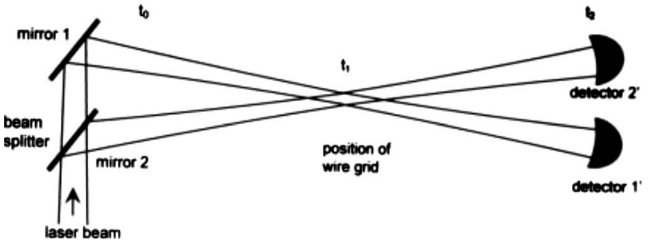

FIG. 3. Modified Afshar experiment.

VI. THE AFSHAR EXPERIMENT AND QUANTUM MECHANICAL INTERPRETATIONS

The Copenhagen interpretation predicts that the wires should intercept the same percent of the light as they do for uniform illumination due to Greenberger–Yasin and BCP. This was shown to be violated in the above discussion.

The relative state formulation of quantum mechanics[20] is sometimes referred to as the many-worlds interpretation.[21] In these models the wave function obeys the empirically derived standard linear model at all times. The observer plays no special role in the theory and therefore there is no collapse of the wave function. The wave function is a real physical quantity that exists in a different real universe. Each measurement causes a decomposition of the wave function into *noninteracting* and *noninterfering* branches, histories, or worlds. In the MWI description of the double slit experiment, a photon detected at detector 1 is in universe M, while the same photon detected at detector 2 is in universe S. The detection of the photon at detector 1 in universe M does not allow for interaction or *interference* with the photon detected at detector 2 in universe S. This does not agree with the results of the Afshar experiment.

In the MWI popularized by Deustsh,[22] he described the double slit experiment slightly differently. When a photon has a choice of two holes to go through, the universe divides in two, and in one world the photon goes one way, while in the other it goes the other way. One brings the two possible paths the photon could have followed back together again at a detector screen, so that they interfere and produce the interference pattern. One has made the two versions of reality fuse back together. They only existed as separate realities during the time that the photon was traveling through the experiment. Again with this version of the MWI, there is a contradiction with the Afshar experiment. An interference pattern was detected during the time that the photon was traveling.

However, the result is fully consistent with the transactional interpretation. The physical standing wave between emitter and absorber in Minkowski space time would be expected to produce interference minima. Therefore, the Afshar experiment gives experimental proof that the transactional interpretation version of the state vector has predictive power over both the Copenhagen interpretation and the many-worlds interpretation.

VII. CONCLUSION

The Copenhagen interpretation correctly refuted a classical phase space state vector, but we may have stumbled on a previously unknown physical quantity, namely, the four-vector standing wave. One can make a cogent argument that the state vector is a standing wave along lightlike paths in Minkowski space time. It requires acknowledging the misnomer of imaginary numbers. It requires examining their inherent timelike nature. It also benefits from awareness that the usefulness of complex numbers has its preeminence with measurements involving time. Most importantly, it requires ontology learned from the Afshar experiment that gives predictive power to the transactional interpretation over other quantum mechanical interpretations. It does not hinge on TI's underdeveloped concept of pseudotime, although it allows for it, if its development becomes clearer.

There has been much debate over the need for visualization in quantum mechanics. Both sides have legitimate points to be made. Those against would say that visualizations are not always successful. One needs only to look at Maxwell's demon[23] or Jaynes'[24] view that entropy is a subjective quantity to see them as unsuccessful. The opposing view would say that this does not mean that visualizations never bear fruit. I would say that visualizations have proven their usefulness. All of Newton's and Einstein's major contributions were developed using this technique. Visualizations are part of the brainstorming process and should not be discouraged—just examined. They can also go beyond simply being a model or a brainstorming activity. They can be correct. Consider this quote.

> To be sure, the [author's] hypotheses' are not necessarily true; they need not even be probable. It is completely sufficient that they lead to a computation that is in accordance with the astronomical observations.

This quote is Osiander's preface to Copernicus's famous heliocentric solar system publication.[25] Stopping at the sufficiency to calculate would have robbed science of one of its crowning achievements. This visualization proved to be invaluably correct.

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